Projectile Instability Produced by Internal Friction

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The stability of a spinning projectile which contains a cylindrical mass fitted loosely into a cylindrical cavity but constrained to spin with the projectile is examined. An unstable coning motion is obtained in which spin decay and cone angle growth are proportional to the coefficient of friction between the mass and the cavity wall and to the maximum angle of cant between the mass and the projectile. Similar behavior is observed experimentally.

Nomenclature

x_s, y_s, z_s	= axes with fixed orientation in space
x, y, z	= non-rolling body axes; z along axis of symmetry
ϕ, θ, ψ	= Eulerian angles
t	= time
$\dot{(\;)}$	$= d(\;)/dt$
p	$= \dot{\psi}$
I	= moment of inertia about axis of symmetry
J	= moment of inertia about transverse axis
$(\;)'$, $(\;)_p$	= refer to projectile
$(\;)''$, $(\;)_b$	= refer to ballast
I_t	$= I_p + I_b$
J_t	$= J_p + J_b$
N_x, N_y, N_z	= components of torque on x, y, z axes
$\omega_x, \omega_y, \omega_z$	= components of angular velocity
D	= diameter of ballast
l	= half-length of ballast
Δr	= radial clearance between ballast and projectile
ζ	= angle defining contact points between ballast and projectile
ξ	$= \pi/4 - \zeta$
T	= torque applied by mechanical coupling to projectile in z' direction
μ, μ'	= coefficients of friction for ballast surface/cavity wall and coupling pin/cavity slot, respectively
a	$= \mu' \cos\xi$
N	= normal force
F	= friction force
γ	$= \lambda - \zeta$
M	$= 2l(N^2 + F^2)^{1/2}$
M_α	= aerodynamic moment derivative
C	= roll damping constant

Introduction

IN the development of gun-fired projectiles it is often desirable to have provision for varying the mass of the round while keeping the aerodynamic shape constant. This is most easily accomplished by placing ballast within the projectile at the center of gravity. To insure full spin, the ballast mass must be engaged to the projectile by a mechanical coupling.

The purpose of this paper is to point out that instability can arise from such an arrangement if radial motion between the ballast and the projectile is permitted. Formulas are derived which relate the growth of coning motion and the roll slowdown to the radial clearance and coefficient of friction between the projectile and the ballast.

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Equations of Motion

Figure 1 shows the sets of axes and defines the Eulerian angles. The direction of flight is along z_s. The axes x, y, z pitch and yaw with, but do not roll with, the "body" (which may be either the projectile or the ballast). The origin of the coordinates is the center of gravity of the body. The angle ψ is included only to clarify the definition of p, which has the same value on projectile and ballast by virtue of the mechanical coupling.

The equations of motion for the body are, in terms of non-rolling axes[1]

$$N_x = J\dot{\omega}_x + I\omega_y\omega_z - J\omega_y(\omega_z - p)$$

$$N_y = J\dot{\omega}_y + J\omega_x(\omega_z - p) - I\omega_x\omega_z$$

$$N_z = I\dot{\omega}_z$$

Angular velocities are related to Euler angle derivatives by[1]

$$\omega_x = \dot{\theta}$$

$$\omega_y = \dot{\phi}\sin\theta$$

$$\omega_z = \dot{\phi}\cos\theta + p$$

Combination of these equations yields, for small θ,

$$N_x = J\ddot{\theta} + I\dot{\phi}\theta\omega_z - J\dot{\phi}^2\theta \tag{1a}$$

$$N_y = J(\ddot{\phi}\theta + \dot{\phi}\dot{\theta}) + J\dot{\theta}\dot{\phi} - I\dot{\theta}\omega_z \tag{1b}$$

$$N_z = I\dot{\omega}_z \tag{1c}$$

Relative Orientation of Ballast and Projectile

Figure 2 shows a cutaway view of the projectile and ballast with a form of mechanical coupling that can be closely approximated analytically. A ring of evenly spaced radial pins is positioned about the equator of the ballast, and these pins engage a series of longitudinal slots in the wall of the cavity within the projectile. Pin length is sufficient that pins cannot disengage from the slots. Slot width is sufficient that the ballast can assume a cocked position in the cavity without the pins binding in the slots.

Figure 3a presents the view in the $-z'$ direction of the ballast within the cavity. The ballast is assumed tilted, contacting the cavity wall at two points defined by ζ. This is taken to be the stable position for the ballast; a general analysis of ballast motion is not required here as a quasisteady solution

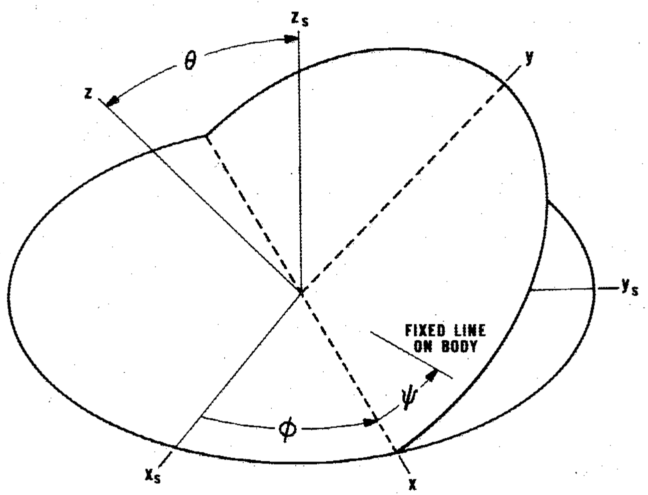

Fig. 1 Coordinate axes.

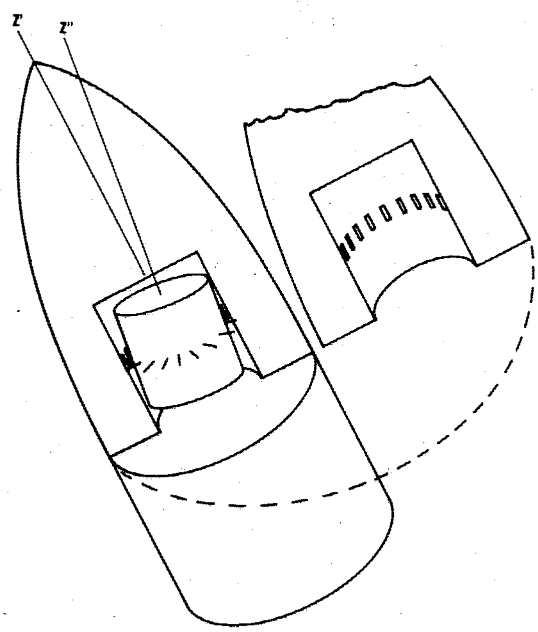

Fig. 2 Cutaway view of projectile with ballast and mechanical coupling.

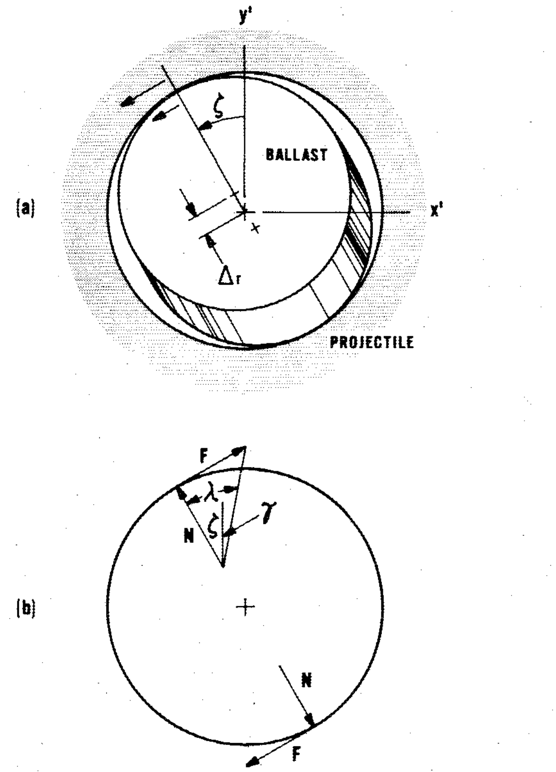

Fig. 3 Normal and frictional loads between ballast surface and cavity wall.

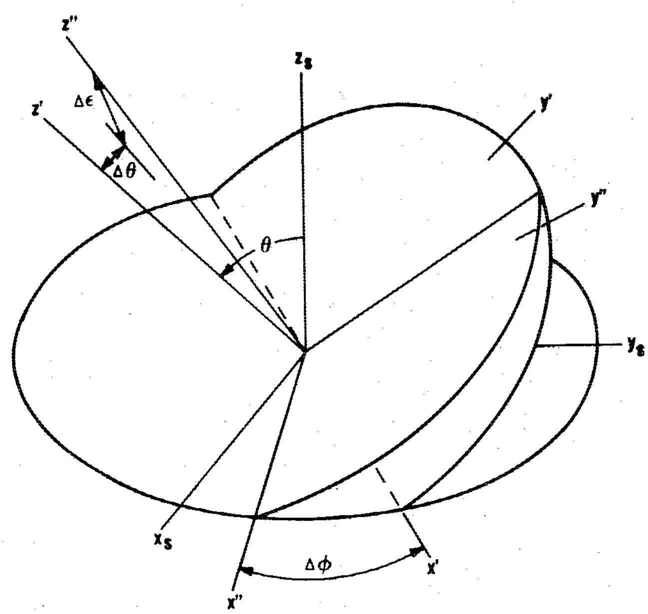

Fig. 4 Relationship between projectile axes x', y', z' and ballast axes x'', y'', z''.

will be sought for the system. The angle ζ will be found to vary only slowly with time.

The clearance Δr and the angle ζ completely define the orientation of the ballast axes with respect to the projectile axes. To transform the former into the latter, one must perform the following rotations:

$$\Delta\theta = (\Delta r/l)\cos\zeta \qquad \text{about } x' \tag{2a}$$

$$\Delta\epsilon = (\Delta r/l)\sin\zeta \qquad \text{about } y' \tag{2b}$$

$$\Delta\phi = (\Delta r/l)(1/\theta_p)\sin\zeta \qquad \text{about } z' \tag{2c}$$

These angles are illustrated in Fig. 4 for a large angle θ. Equations (2a-2c) are valid only for θ small and $\Delta r/l \ll \theta$, conditions which will be satisfied in applications discussed later.

Transformation of torque components between the projectile and ballast axes is specified by

$$\begin{Bmatrix} N_{x''} \\ N_{y''} \\ N_{z''} \end{Bmatrix} = \begin{bmatrix} 1 & -\Delta\phi & \Delta\epsilon \\ \Delta\phi & 1 & -\Delta\theta \\ -\Delta\epsilon & \Delta\theta & 1 \end{bmatrix} \begin{Bmatrix} N_{x'} \\ N_{y'} \\ N_{z'} \end{Bmatrix} \tag{3}$$

Loads on Ballast and Projectile

Figure 3b illustrates the forces, normal and frictional, applied to the cavity wall by the ballast at the two points of contact. The ballast is driven by the mechanical coupling at the same rotational speed as the projectile. Since the cavity has a larger radius than the ballast, there must be a difference in peripheral velocity of the two bodies at the points of contact. As a result, the friction forces F are directed so as to slow the rotation of the projectile.

While it tends to slow the projectile, cavity wall friction acts to accelerate the ballast. This action must be opposed by reactions between the pins and slots of the coupling. Now it

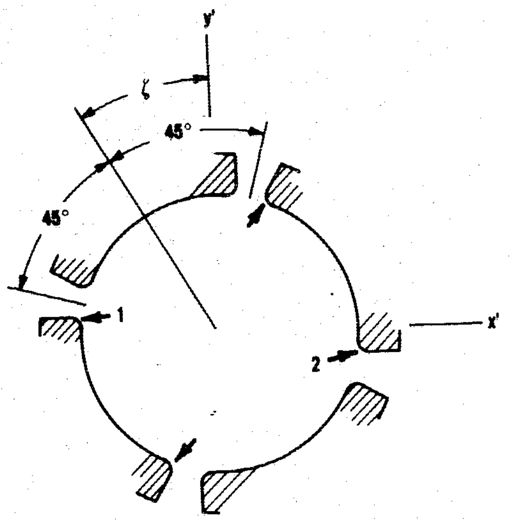

Fig. 5 Points of minimum clearance between coupling pins and slot surfaces. Ballast is assumed tilted in direction defined by ζ. For clarity, slot width is exaggerated and only four slots are shown.

can be shown that, for finite tilt angle $\Delta r/l$, pin/slot clearance is minimum at positions which lie ± 45 deg away from the direction of tilt defined by ζ. These points of minimum clearance are shown in Fig. 5. Under the action of the wall friction then, the ballast will rotate sufficiently to allow the necessary reactions to be provided by two diametrically opposed slots at 45 deg to the tilt direction. For the direction of rotation assumed in Fig. 3, the ballast tends to be accelerated counterclockwise (CCW) by the wall friction. Hence the reaction torque on the ballast must be clockwise (CW), and the pair of slot surfaces 1 and 2 in Fig. 5 will provide the reactions.

The forces applied by the pins to surfaces 1 and 2 will be equal in magnitude but opposite in sense. Because there is relative motion between each pin and the surface it contacts, each force will have longitudinal and radial components, R_l and R_r, in addition to the circumferential component R_c. From consideration of pin/slot kinematics, it can be shown that the radial slip velocity is much smaller than the longitudinal velocity; hence R_r is negligible, and $R_l \approx \mu' R_c$. In Fig. 5, for CCW rotation, the pin at point 1 would rise out of the page relative to the slot surface, while that at point 2 would descend. Thus R_l on surface 1 is directed out of the page and R_l on surface 2 into the page. The result is that, in providing the necessary reactions to wall friction, the coupling applies a torque, say T, to the projectile in the z' direction and another torque, $\mu' T$, in the x'-y' plane, oriented 45 deg CW from the direction of tilt.

This analysis has tacitly assumed an infinite number of pins and slots, so that there is always a pair available at the position of minimum clearance. For a finite number of pairs n, each pair will provide the reactive torque for $1/n$ of each revolution, and as a consequence the torque component $\mu' T$ will oscillate in the x'-y' plane through a range of π/n radians about the ideal direction.

All loads between the projectile and the ballast in this system consist of pure torques, since the centers of gravity of the two bodies coincide. In total, the moments applied to the projectile are, for small γ,

$$N_{x'} = -M + M_\alpha \theta_p + \mu' T \sin\xi \tag{4a}$$

$$N_{y'} = M\gamma + \mu' T \cos\xi \tag{4b}$$

$$N_{z'} = T - FD - C\omega_{z'} \tag{4c}$$

The quantity μ' will be assumed to be sufficiently small so that the associated term in Eq. (4a) can be neglected; the validity of all the assumptions made in the analysis will be discussed in the appendix. Deleting the aerodynamic terms and multiplying the foregoing moments by -1 yields the moments applied to the ballast by the projectile. Transformed to ballast axes via Eq. (3), these become

$$N_{x''} = M + (M\gamma + aT)\Delta\phi + (FD - T)\Delta\epsilon \tag{5a}$$

$$N_{y''} = M\Delta\phi - M\gamma - aT + (T - FD)\Delta\theta \tag{5b}$$

$$N_{z''} = -M\Delta\epsilon - (M\gamma + aT)\Delta\theta - T + FD \tag{5c}$$

Also

$$\theta_b = \theta_p - \Delta\theta \tag{6a}$$

$$\phi_b = \phi_p - \Delta\phi \tag{6b}$$

$$\omega_{z''} = \omega_{z'} + \dot{\phi}_b \cos\theta_b - \dot{\phi}_p \cos\theta_p \tag{6c}$$

Approximate Model

Flight tests of instrumented projectiles with "loose" ballast ($\Delta r/l \approx 10^{-3}$) have shown a characteristic departure from the behavior of standard rounds. Normal flight involves a coning motion of limited amplitude, (a few degrees maximum) at precession frequencies defined by the inertial properties of the shell and by the aerodynamic moment derivative.[2] Also, aerodynamic drag produces deceleration in roll. Ballasted rounds, on the other hand, show a steady, unlimited growth in cone angle θ and a roll deceleration $-\dot{\omega}_z$ much higher than normal.

To obtain a model that describes this unstable behavior in the simplest terms, the preceding equations will be simplified by discarding terms shown to be small by flight tests. On the basis of observed behavior, a quasisteady response is sought, and the accelerations $\ddot{\theta}$ and $\ddot{\phi}$ will be neglected, as will the quantity $\dot{\zeta}$. Also, the angle γ will be assumed to be sufficiently small so that $\zeta \approx \lambda$. Typical magnitudes of various quantities from experimental trials are listed below. The value of θ corresponds to a cone angle at which the anomalous behavior might become evident.

I_b/J_b	$=1$	θ	$=5\times 10^{-2}$
J_t/J_b	$=50$	$\Delta\theta$	$=10^{-3}$
I_t/I_b	$=5$	$\Delta\epsilon$	$=10^{-3}$
J_t/I_t	$=10$	$\Delta\phi$	$=10^{-2}$
J_p/I_p	$=49/4$	$\dot{\theta}/\omega_z$	$=5\times 10^{-5}$
$\Delta r/l$	$=10^{-3}$	$\dot{\phi}/\omega_z$	$=10^{-1}$
$M_\alpha/J_t\omega_z^2$	$=10^{-3}$		
$C/I_t\omega_z$	$=10^{-5}$		

If only the dominant terms are retained, the following expressions can be derived from the equations indicated in parentheses. Primes and subscripts have been omitted where distinction between ballast and projectile is no longer necessary.

(2, 6)

$$\dot{\theta}_b = \dot{\theta}_p - \dot{\gamma}\Delta\epsilon \tag{7a}$$

$$\dot{\phi}_b = \dot{\phi}_p + (\dot{\theta}/\theta)\Delta\phi \tag{7b}$$

$$\omega_{z''} = \omega_{z'} + \dot{\phi}\theta\Delta\theta + (\dot{\theta}/\theta)\Delta\phi \tag{7c}$$

$$\dot{\omega}_{z''} = \dot{\omega}_{z'} + \dot{\phi}\dot{\theta}\Delta\theta - (2\dot{\theta}^2/\theta^2)\Delta\phi \tag{7d}$$

(1c, 4c, 5c, 7d)

$$FD - T = \frac{M\Delta\epsilon + aT\Delta\theta - C\omega_z I_b/I_p}{1 + I_b/I_p} \tag{8}$$

$$\dot{\omega}_z = -(M\Delta\epsilon + C\omega_z + aT\Delta\theta)/I_t \tag{9}$$

(1a, 5a)

$$M = I_b\dot{\phi}_b\theta_b\omega_z - J_b\dot{\phi}_b^2\theta_b \tag{10}$$

(2b, 9, 10)

$$-\dot{\omega}_z = \frac{C\omega_z}{I_t} + \frac{\dot{\phi}\theta}{I_t}(I_b\omega_z - J_b\dot{\phi})\frac{\Delta r}{l}\frac{\mu}{\sqrt{1+\mu^2}}\left(1 + \frac{aD}{2l}\frac{1}{\sqrt{1+\mu^2}}\right) \tag{11}$$

(1a, 4a, 10, 6, 7)

$$\dot{\phi}^2 - \frac{I_t}{J_t}\omega_z\dot{\phi} + \frac{1}{J_t}\left[M_\alpha + \frac{\Delta r}{l}\frac{\dot{\phi}}{\theta}\frac{1}{\sqrt{1+\mu^2}} \times \left(I_b\omega_z - J_b\dot{\phi}\right)\right] = 0 \tag{12}$$

(1b, 2c, 4b, 5b, 10, 13)

$$\dot{\theta} = \frac{\Delta r}{l}\frac{\mu}{\sqrt{1+\mu^2}}\frac{I_b\omega_z - J_b\dot{\phi}}{2J_t - I_t\omega_z/\dot{\phi}} \tag{13}$$

$$\gamma = \frac{\Delta r}{l}\frac{\mu}{\sqrt{1+\mu^2}}\frac{1}{\theta}\frac{2J_p - I_p\omega_z/\dot{\phi}}{2J_t - I_t\omega_z/\dot{\phi}} - \frac{a}{2}\frac{D}{l}\frac{\mu}{\sqrt{1+\mu^2}} \tag{14}$$

Equation (10) can be simplified to

$$M = \dot{\phi}\theta(I_b\omega_z - J_b\dot{\phi}) \tag{15}$$

Discussion

Equations (11, 12, and 13), which describe the effects of clearance and friction on projectile behavior, are the principal results of the analysis. Appearing in all these equations is the term $A = I_b\omega_z - J_b\dot{\phi}$. For the preceding set of typical parameters in experimental trials, it is seen that $A > 0$; only this case will be discussed here. (When $A = 0$, the ballast and projectile precess together without interaction, and the quantities N, F, and M are zero.)

Equation (12) differs from the conventional coning rate expression by having a ballast-related term that adds to the destabilizing effect of aerodynamic moment. Denoting the complete expression within brackets by B, we note that steady precession is possible only if $4B < I_t^2\omega_z^2/J_t$, the condition for gyroscopic stability.[3] Under this condition there are two solutions for $\dot{\phi}$, one that approaches zero if $B \to 0$ and another that approaches $\omega_z I_t/J_t$. (It is clear from the position of θ in Eq. (12) that this analysis does not apply for arbitrarily small cone angles. Rather, it describes the growth or subsidence of a previous disturbance.)

Equation (11) gives the roll deceleration of the projectile. Since $A > 0$, the presence of the ballast adds to the aerodynamic spin decay. Equation (11) is the only expression among the results that contains the coupling friction coefficient μ'. The contribution of μ' is rather small, about 20% of the total ballast-related contribution for $\mu = \mu' = 0.25$ and $D/2l = 1$. This small effect of μ' suggests that the response may not be sensitive to 1) the variation in direction of the coupling friction torque caused by a finite number of pins and slots, and 2) the specific type of coupling used.

Equation (13) gives the cone opening rate $\dot{\theta}$. The difference term in the numerator is positive, while that in the denominator is negative for the smaller root $\dot{\phi}$ of Eq. (1) but positive for the larger root. Thus, the slower precession mode decays with time whereas the faster mode is associated with cone angle growth or dynamic instability.[3]

In the most general case it is not possible to identify a single parameter that effectively controls the instability, although from Eqs. (11) and (13) it is clear that $\dot{\theta}$ and the anomalous roll deceleration are both proportional to $\mu\Delta r/l$. Simplifying the model to the maximum extent (the approximations involved are readily deduced from inspection of Eqs. (11-13)), we obtain

$$\dot{\phi} = \omega_z I_t/J_t\,, \quad -\dot{\omega}_z = C\frac{\omega_z}{I_t} + \left(\mu\frac{\Delta r}{l}\frac{I_b}{J_t}\right)\theta\omega_z^2\,, \quad \dot{\theta} = \left(\mu\frac{\Delta r}{l}\frac{I_b}{J_t}\right)\omega_z$$

In simplest terms, then, the severity of the instability is indicated by the term $\mu(\Delta r/l)(I_b/J_t)$, and this quantity should be minimized, within the constraint of fixed mass of ballast, to minimize the destabilizing effect.

This instability places very stringent requirements on the clearance between ballast and cavity. Consider the following parameter values:

$\mu = 0.25$ $\quad l = 50$ mm $\quad I_b/J_t = 0.1$

$\Delta r = 0.05$ mm $\quad$ (0.002 in. clearance)

The preceding equation for $\dot{\theta}$ yields, for $\omega_z = 100$ rev/s, a cone opening rate of approximately 1.0 deg/s. For a large projectile with a flight time of perhaps 20 s, this would be unacceptable.

Appendix: Approximations

The various approximations used in the analysis will now be discussed. First, the angle γ was considered sufficiently small that $\sin\gamma \approx \gamma$ and $\zeta \approx \lambda$. For typical values of $\Delta r/l\theta$, γ is dominated by the coupling-friction term in Eq. (14), and that term will suffice for estimating magnitude

$$\gamma \approx -\frac{\mu\mu'}{\sqrt{1+\mu^2}}\frac{D}{2l}\sin(\pi/4 - \lambda + \gamma)$$

To provide an impression of the range in μ and μ' permitted by the two foregoing approximations, the expression for γ has been used, with $D/2l = 1$, to find combinations of μ and μ' that give 1) $-0.1 < \gamma < 0$ and 2) $-0.1\ \lambda < \gamma < 0$, or $\lambda < \zeta < 1.1\ \lambda$.

The first condition is found to be satisfied for $0 \le \mu \le 1$ if $\mu' < 0.9$. The second is satisfied if μ' does not exceed the following values for various μ:

μ	maximum μ'
0.0	0.14
0.2	0.19
0.4	0.29
0.6	0.55
0.8	2.50

It is clear that the second condition will usually be the more restrictive, and that low friction at the ballast/cavity interface must be accompanied by low friction in the coupling if the analysis is to apply.

In Eq. (4a), the quantity $\mu' T\sin\xi$ was neglected relative to M. Now Eq. (8) shows that the reaction torque T from the mechanical coupling is essentially equal to FD

$$T = FD = (D/2l)M\mu/\sqrt{1+\mu^2}$$

Thus

$$\frac{\mu' T\sin\xi}{M} = \frac{\mu\mu'}{\sqrt{1+\mu^2}}\frac{D}{2l}\sin(\pi/4 - \lambda + \gamma)$$

But this is equal to $-\gamma$, and the earlier remarks for small γ are relevant.

References

[1]Goldstein, H., *Classical Mechanics*, Addison-Wesley Press, Cambridge, Mass., 1951.

[2]Chadwick, W. R., "Dynamic Stability of the 5"/54 Rocket-Assisted Projectile (The Influence of a Nonlinear Magnus Moment)," Naval Weapons Laboratory, Dahlgren, Va., TR-2059, Oct. 1966.

[3]Murphy, C. H., "Free Flight Motion of Symmetric Missiles," Ballistic Research Laboratories, Aberdeen, Md., TR-1216, July 1963.